Calculation methodology for stiffness coefficient of the ground under the railway subgrade

The article presents calculation methodology for stiffness coefficient of the ground under the railway subgrade. The preparation of numerical models and the determination of the stiffness coefficient using numerical analyses are discussed. The relationship between generating vibrations in the function of the structural cross-section of the railway subgrade and material parameters of individual foundation layers is presented.

Keywords: stiffness coefficient, railway subgrade.

Wstęp

Each elastic medium is deformed due to the load acting on it depending on the type and value of forces (concentrated, continuous, pulsed, one-off, harmonic, periodic or stochastic load, etc.). The railway subgrade system of rail vehicles is one of them. Solid structures (rocks) are relatively little deformed, but they are deformed intensively, quickly. These deformations are practically entirely reversible (if they remain within the scope of elastic deformations and if they do not cause the medium loses its continuity). Fragmented media (cracked rocks, soils) that include the subgrade, are deformed significantly and relatively slowly, depending on the level of fragmentation, porosity and medium coherence as well as the values of forces acting on them. These deformations are only partially reversible. Large pores between grains and internal forces that significantly affect the dissipation of energy transferred directly from the rail vehicle are specific features for the disintegrated medium. Since grains and particles are irregularly arranged, some elements are more loaded when strictly compressed. This is why grains and particles permanently move toward each other, and thus, deformations of the disintegrated medium are non-linear and most often irreversible. It should also be remembered that where points of grains or particles are in contact, design stresses related to the entire cross-section of the soil are considerably higher. In soil media, there is no linear relationship between deformations and stresses. To distinguish soil deformation parameters from elastic bodies, the following features were introduced:

1. modulus of elasticity $E$ – under conditions of uniaxial compression and free lateral expansion of the soil,
2. modulus of volume elasticity (oedometric) $M$ – under conditions of uniaxial compression, with no possibility of lateral expansion of the soil sample.

With repeated loads and relieves, the soil deformation curves are reproducible and parallel to each other - it can therefore be assumed that there is a certain elasticity of the soil.

The issue which is the subject of the article is to determine the stiffness coefficient of the ground under the railway subgrade of the rail vehicle. The existing experimental methods are based on laboratory tests and consist in the determination of the Young’s E modulus and the oedometric module M, which is also a Young’s modulus determined with some restrictions that do not allow for volumetric expansion. Hence the idea to determine the stiffness of the railway subgrade using numerical analysis, as a system approach to rail transport routes and their dynamic impact on the environment and the selection of vibroisolation systems.

The purpose of the work is to present the results of analyses of a model of the global propagation to the environment of vibrations created as a rail vehicle passes and possible influence of the vibration wave propagation on engineering structures, e.g. residential buildings. The work also presents the impact of the railway subgrade vibration isolation system on the minimisation of vibrations. The model domain used for the analysis includes a railway subgrade along with the building, which was selected as the building that is most exposed to the dynamic impact of passing rail vehicles. The work includes a description of the model, boundary conditions and loads adapted to the conditions prevailing in the Silesian agglomeration. The loads were adopted on the basis of real measurements that took place in the Silesian agglomeration. The results of numerical analyses of the model described were referred to the PN-85/B-02170 Standards (Evaluation of Harmfulness of Vibrations Transmitted to Buildings by the Ground) PN-B-02170: 1985 - Harmfulness of Vibrations Transmitted to Buildings by the Ground and PN-88/B-02171 (Evaluation of the Impact of Vibrations on People In Buildings) [5]. PN-B-02171:1988 - Assessment of the Impact of Vibrations on People in Buildings aimed to ensure adequate comfort for residents and to protect buildings against damage such as cracks. This was the basis to formulate a global model to determine soil stiffness using the finite element method, prepared in the LS-Dyna program [2,3], which is a leading application using integration by explicit method.

The paper’s scope of research is:
1. carrying out measurements in a selected section of a tram railway subgrade in the Silesian agglomeration,
2. development of a numerical model allowing for full analysis of dynamic effects in any chosen location,
3. preparation of boundary conditions and loads based on measurements that can be carried out anywhere; the article is based on measurements carried out in the Silesian agglomeration,
4. carrying out comparative simulations of vibration wave propagation measurements in the case of Prefa track systems without vibroisolation and with vibroisolation using elastomer plates produced by FFT.
5. simulation of the influence of the propagating vibration wave on the architectural structure (building) and the people inside it.

The article presents a description of the adopted model and guidelines for the model in order to conduct numerical analysis of the impact of vibroisolation on vibrations transmitted to engineering structures (buildings) that can be used in any agglomerations in which rail transport (tram and railway) is operated, as well as to assess the impact of car transport in any geotechnical conditions.

1. Vibration measurements

Measurements of ground vibrations were carried out on a tram-road crossing located in Będzin, Poland, at Aleja Hugona Kollatja (Figure 1).
In order to collect data for forcing vibrations in the Silesian agglomeration the measurements were taken at the crossroads of the 910 provincial road and ulica Świerczewskiego (Świerczewskiego Street) in Będzin, Poland [1]. Ground properties in this region are similar to those in the analysed location (Silesian location), therefore the results of measurements that were performed in order to assess dynamic impacts generated on a road-tram crossing operated in Tramwaje Śląskie S.A. Company can be applied as well. In figure 2 the location of the conducted vibration measurements and the coordinate system adopted is presented.

The following measuring equipment was used during the tests:

6. TEAC Gx-01 signal recorder,
7. measuring computer,
8. 4 pcs. of PCB 356A16 three-axis piezoelectric accelerometers,
9. 3 pcs. of PCB 393A03 uniaxial piezoelectric seismic accelerometers.

At all measuring points, vibration time courses were recorded using PCB 356A16 three-axis piezoelectric accelerometers mounted on steel disks located on the ground. In addition, at points located directly at the tram railway subgrade, the vibration time courses were recorded using uniaxial PCB 393A03 piezoelectric accelerometers in the vertical direction. The tests were carried out in a grid of 4 measuring points arranged in accordance with figure 3.

10. determining the spectra of the signals recorded.

Examples of time courses registered can be found in the figures below. The load to the model was used as an acceleration in a location that corresponds as accurately as possible to the measurement scenario. Figure 4 presents examples of time courses of the measured acceleration amplitudes, while figure 5 presents the amplitude-frequency course.

Measurement data were pre-processed by applying filters to the signals recorded. The scope of the tests conducted included:

10. recording time courses of ground vibrations as the trams pass,
11. determining the maximum amplitude of ground vibration acceleration for each point and measuring direction

2. Simulation model of a tram railway subgrade

2.1. Location of the Numerical Cross-Section Tested

The numerical model developed has been adapted to imitate conditions in each agglomeration, in particular in Silesia. As an example, the currently modernised section in the area of Chorzowska Street in Katowice, Poland, has been used. Since the distance between the buildings and the tram subgrade line is critical due to dynamic impacts as a result of passing trams that cause vibrations, which in turn are transferred to buildings. The area of the Chorzowska Street intersection along with Złota Street has been selected for the analysis. Its computational domain that contains the eastern corner of the intersection with the tram subgrade and the building in the immediate vicinity of tram tracks is shown in figure 6. There is a building in the immediate vicinity of tram tracks (8 m from the track axis) in this location.
2.2. Numerical Model Grid

In order to simulate the propagation of vibrations from the tram railway subgrade, its calculation model was adopted, in the form of a grid, which maps the typical location of the tram railway subgrade in the urban agglomeration, which is shown in figure 7. The model takes into account different types of ground and a building in the vicinity of tram tracks.

Fig. 7. Detailed grid of the numerical model of the tram railway subgrade

The integration time has been selected so that the Courant criterion is kept with the mass scaling as small as possible (<2.5%). In addition, the maximum length of the element has been determined to correctly reflect the wave phenomena at higher frequencies.

2.3. Material calculation data for the numerical model

Soil material data has been developed on the basis of geological research. Due to the relatively large differences in the soil layers between the individual measuring points, the average stiffness values of the material were used for the calculation. Damping and stiffness coefficients used in the analyses were adopted on the basis of „Structural Damping Values as a function of dynamic response stress and deformation levels” - J.D. Stevenson and „Vibroisolation Systems in Rail and Car Transport” - J.Targosz [6].

2.4. Boundary Conditions for the Numerical Model

Since geomechanical analyses of the subsoil make the computational domain virtually unlimited, in the numerical model it should be limited to achieve rational calculation times. For this purpose, special boundary conditions have been used, which do not reflect waves – PML model (ang. Perfectly Matched Layer) according to the assumptions presented in figure 8.

Fig. 8. Scheme of the PML boundary condition operation

By applying specialized boundary conditions based in figure 8, the results of calculations will not be disturbed by phenomena such as reflection or wave interference. In figure 9 presented below, the computational domain with the boundary conditions of the simulation is illustrated.

Fig. 9. Boundary conditions used in the numerical model of the railway subgrade

2.5. Forcing vibrations - loads adopted as an input parameter for simulation

In order to collect data for forcing vibrations in the Silesian agglomeration. The measurements were taken at the crossroads of the 910 provincial road and ulica Świerczewskiego (Świerczewskiego Street) in Będzin, Poland, ground properties in this region are similar to those in the analysed location (Silesian location), therefore the results of measurements that were performed in order to assess dynamic impacts generated on a road-tram crossing operated in Tramwaje Śląskie S.A. Company can be applied as well.

3. Results of numerical calculations

The results of numerical analyses are presented in the form of maps of acceleration distributions for various time moments. Below, in figures 10 and 11, iso-surfaces for selected moments of time for the solution without vibroisolation and with vibroisolation are presented.

Fig. 10. Iso-surfaces of wave propagation - distribution of acceleration resultant [mm/s²] - model without vibroisolation

Fig. 11. Iso-surfaces of wave propagation - distribution of acceleration resultant [mm/s²] - model with vibroisolation

As in the case of the iso-surface, in figure 12 below, the results of calculations in vertical planes for the model without vibroisolation and for the model with vibroisolation were presented.
According to experience, the propagation of waves in the ground caused by means of transport usually takes the form of a Rayleigh wave (interference of a cutting wave with a longitudinal wave) - i.e. a surface wave in which matter particles make a circular motion. For the measuring point presented in figure 13, the trajectory of the particle motion was determined, which is presented in figure 14.

The trajectory presented confirms the occurrence of Rayleigh wave on the surface of the ground; this is one of the premises that indicate that computational task was conducted correctly.

In order to better visualise the vibrations caused by the passing tram vehicle, their impact on the vibrations of the building structure located near the tram track was presented. In figure 15 below, measurement points were presented, while in figures 16 and 17, the time courses of accelerations in rigid building nodes on the side of vibrations forced were presented for the model with vibroisolation and the tram railway subgrade set on concrete.

The above courses were used to analyse and evaluate the impact of vibrations on buildings according to PN-85/B-02170. Spectra and spectrograms for the above signals are presented in the following figures 18 and 19.
The results of these analyses were used to assess the threat of building vibration according to the SWD scale for a model with a vibroisolation system and without a vibroisolation system as presented in figure 20.

Fig. 20. The results of the numerical analysis determined for building nodes placed in the SWD scales

4.1. The construction of the subgrade

In order to carry out a simulation of railway subgrade vibration propagation in the ground, a computational model was adopted that represents the location of the rail vehicle subgrade, the construction of its subgrade and a geological structure, e.g. for the area of Upper Silesia Area, was presented in figure 21.

Fig. 21. Geological map of Poland - the area of the Silesian Agglomeration at a scale of 1: 500000 [4]

A mesh of finite elements has been prepared so that it can be adapted to any other railway subgrade system solution without and with vibroisolation with a relatively low workload. In figure 22, a typical cross-section of the railway subgrade with its structural components was presented.

Fig. 22. Cross-section of rail vehicle subgrade

4.2. Numerical model grid

In order to carry out a simulation of railway subgrade vibration propagation in the ground, a computational model in the form of a grid was adopted, which maps the typical location of the rail vehicle subgrade presented in figure 23. The model may take into account different types of ground.

Fig. 23. Detailed grid of the numerical model of a tram subgrade

The integration time has been selected so that the Courant criterion is kept with the mass scaling as small as possible (<2.5%). In addition, the maximum length of the element has been determined to correctly reflect the wave phenomena at higher frequencies.

4.3. Material calculation data for the numerical model

Soil material data has been developed on the basis of geological research. Due to the relatively large differences in the component soils between the individual measuring openings, the average stiffness values of the material were adopted for the calculation. Journal of Laws of 1999 No. 43, item. 430 contains the material data of road pavements used for calculations. Damping and stiffness used in the analyses were adopted on the basis of "Structural Damping Values as a function of dynamic response stress and deformation levels" - J.D. Stevenson and "Vibroisolation Systems in Rail and Car Transport" - J. Targosz. The results are summarized in table 1.
Tab. 1. Material data used in numerical calculations

<table>
<thead>
<tr>
<th></th>
<th>Vibrator isolation washer</th>
<th>Granite crushed stone</th>
<th>Key aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density [t/mm³]</td>
<td>1.20E-09</td>
<td>2.0E-09</td>
<td>2.20E-09</td>
</tr>
<tr>
<td>Critical damping coefficient</td>
<td>0.05</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.49</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Oedometric modulus [MPa]</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Young's modulus [MPa]</td>
<td>7.2</td>
<td>200</td>
<td>12900</td>
</tr>
<tr>
<td>Speed of sound [mm/s]</td>
<td>3.204E+05</td>
<td>3.187E+05</td>
<td>2.552E+06</td>
</tr>
</tbody>
</table>

Fig. 24. Model support diagram

4.4. Boundary conditions for the numerical model

The boundary conditions adopted for the numerical model that were identical to those in step 2.4. of this work. By applying specialised boundary conditions based in figure 8., the results of calculations will not be disturbed by phenomena such as wave reflection or wave interference. Figure 24 included below presents the computational domain with the boundary conditions of the simulation. The model was supported on each surface in the normal direction to the surface that limits the computational domain. The colours correspond to the axes of the coordinate system towards which the degrees of freedom have been removed.

Fig. 25. Model load diagram

5. Results of numerical calculations

The results of numerical analyses are presented in the form of the distribution of displacements (deformations) in figure 26.

Fig. 26. Distribution of amplitudes of ground displacements under the railway subgrade

The results of the calculation of the basic parameters for determining the volumetric stiffness of the subsoil under the railway subgrade are presented in table 2.

Tab. 2. Data for numerical calculations and results of numerical calculations of basic ground parameters necessary to determine stiffness

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axle Load</td>
<td>N</td>
<td>120</td>
<td>kN</td>
<td>Assumed Value</td>
</tr>
<tr>
<td>Plate Size</td>
<td>A</td>
<td>0.6</td>
<td>m</td>
<td>Manufacturer's Data</td>
</tr>
<tr>
<td>Plate Size</td>
<td>B</td>
<td>2.2</td>
<td>m</td>
<td>Manufacturer's Data</td>
</tr>
<tr>
<td>Young's Modulus of the Soil</td>
<td>E</td>
<td>70000</td>
<td>kPa</td>
<td>Reference Data</td>
</tr>
<tr>
<td>Coefficient</td>
<td>alfa</td>
<td>0.9</td>
<td>[-]</td>
<td>Data for Clay</td>
</tr>
<tr>
<td>Soil Weight by Volume</td>
<td>gamma</td>
<td>18</td>
<td>kN/m³</td>
<td>Reference Data</td>
</tr>
<tr>
<td>Plate Load</td>
<td>sigma</td>
<td>91</td>
<td>kPa</td>
<td>Formula 1</td>
</tr>
<tr>
<td>Extent of Ground Displacements</td>
<td>Z_o</td>
<td>0.68</td>
<td>m</td>
<td>Formula 2</td>
</tr>
</tbody>
</table>

Then, using the relationship for stresses in the following form:

\[ \sigma = \frac{N}{A \times B} \]  

(1)

and for the extent of ground displacement according to the relationship presented below:

\[ Z_o = \frac{a}{\ln(z_o)} \times \frac{\sigma}{y} \]  

(2)
the coefficient of volumetric stiffness was determined according to the relationship:

\[ k = \frac{\sigma}{s} \]  

(3)

where: \( s \) – maximum amplitude, \( s = 0.0088 \text{ m} \).

Hence, the value of the volumetric stiffness coefficient of the ground under the railway subgrade is equal to \( k = 113 \text{ MN/m}^3 \).

Distribution and depth of ground displacements penetration are presented in figure 27.

![Distribution and depth of ground displacements penetration](image)

**Fig.27.** Distribution and depth of ground displacements penetration

**Summary**

Based on the vibroisolated substructure numerical simulation that was carried out, it can be concluded that on the basis of numerical analysis the stiffness of the ground under the rail vehicle subgrade can be determined. The numerical model presented should be verified by way of experiments and this may be the basis for the design of railway subgrade with limited dynamic impact.

In addition, based on the simulations carried out with the numerical model of the vibroinsulated subgrade and the subgrade without vibroisolation, it can be concluded that:

1. With the use of vibroinsulating mats, the propagation of vibrations transmitted by the ground to building structures adjacent to the railway subgrade track can be significantly reduced.
2. Analysis of spectrograms demonstrated that with the use of vibroinsulating mats, vibrations can be reduced in a wide spectrum of frequencies relevant to a building.
3. The level of vibrations transmitted through the ground to building structures using a vibration isolation mat will be reduced by approximately 30% for the applied forced signal.
4. According to PN-B-02170:1985 Standard – Assessment of harmfulness of vibrations transmitted by the ground to buildings - vibrations are present in the most desirable zone, i.e. with no impact on the building. At the same time, it should be emphasised that the analysis was carried out for a specific course of forced vibrations. It is not possible to analyse all possible signals produced by passing rail vehicles.
5. Analysis of the trajectory of nodes on the surface of the ground demonstrated that the ellipsoidal trajectory is specific, which is an evidence that Rayleigh wave was created, which is characteristic for this type of forced vibrations.

At the same time, it should be emphasised that the analysis was carried out for a specific course of forced vibrations. It is not possible to analyse all possible signals produced by passing rail vehicles.

The results of numerical tests should be verified on the actual structure using correctly selected vibroisolation, as presented in the first part of this article.

**Bibliography:**

2. LS-DYNA Theory Manual
4. Geological map of Poland 1:500 000.
5. PN-B-02170:1985 - Assessment of Harmfulness of Vibrations Transmitted by the Ground to Buildings.

**Metodologia obliczania współczynnika sztywności gruntu pod podtorzem kolejowym**

W artykule przedstawiono metodę obliczania współczynnika sztywności gruntu pod podłożem kolejowym. Omówiono przygotowanie modeli numerycznych i wyznaczenie współczynnika sztywności za pomocą analiz numerycznych. Przedstawiono związek między generowaniem drgań w funkcji przekroju strukturalnego podłoża kolejowego a parametrami materiałowymi poszczególnych warstw fundamentowych.

**Keywords:** współczynnik sztywności, podłożo kolejowe.

**Authors:** współczynnik sztywności, podłożo kolejowe.

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